# Approximating prior-based methods in a ROI

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EXTREMA 2015 - Challenges in X-Ray Tomography
“Large” objects exceeding the detector field of view (region-of-interest tomography)

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# Introduction

- We want to exploit prior knowledge in problems with truncated data
    - TV-MIN, wavelet regularization, ...
- Improve reconstruction quality with few projections / noise
- Might reduce truncation artefacts as well!
- Problem: Simply applying standard prior-based methods to ROI tomography does not work

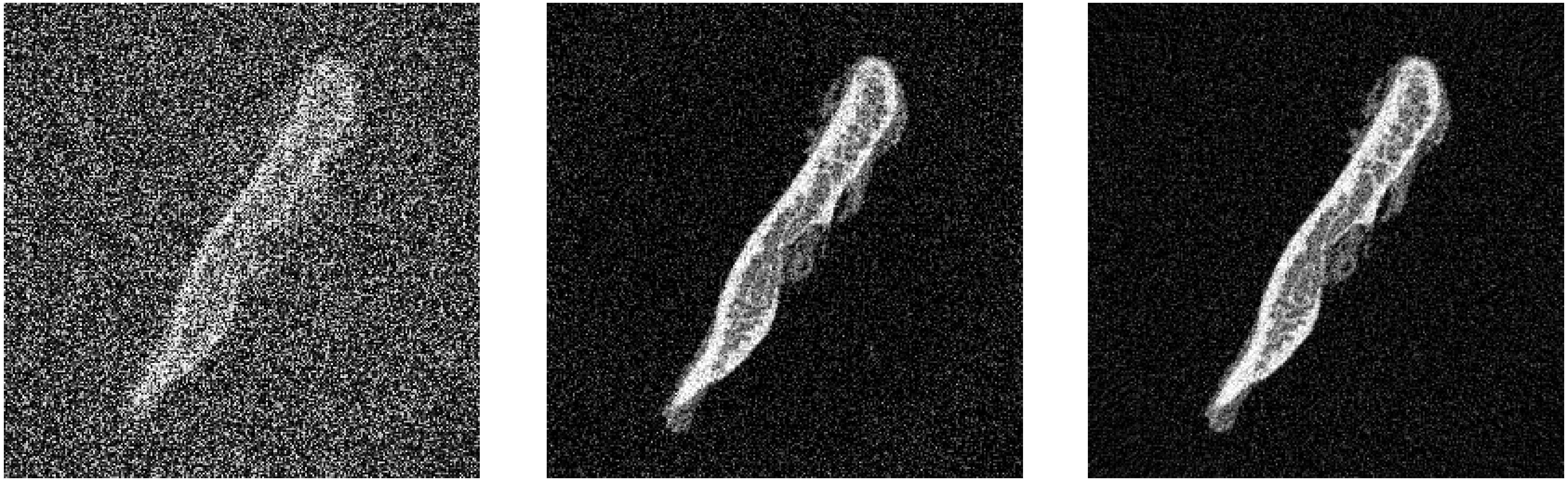
# Prior-based methods and ROI tomography

- Most prior-based methods are algebraic and need full volume during reconstruction
- Simply extending the volume to include region outside ROI leads to problems:
    - Computation time / memory requirements
    - (Greatly) increases number of unknowns without increasing data
    - Parts of volume only influence few projections (or only 1!)
- Filtered backprojection with padding is commonly used at the moment
- Proposal: extend recent filter-based methods to add ability to exploit prior knowledge

# Filter based methods

- Recently developed filter-based methods aim to approximate algebraic method by filtered backprojection (FBP)
- Usually compute a special filter that is geometry and/or data dependent

1024 x 1024 pixels, 256 projections, Poisson noise



(a) FBP (b) SIRT (c) SIRT-FBP

(Pelt & Batenburg, to appear 2015)

# Approximate algebraic method

- Take the standard equation for the algebraic SIRT method:

$$\boldsymbol{x}^{i+1} = \boldsymbol{x}^i + \alpha \boldsymbol{W}^T(\boldsymbol{p} - \boldsymbol{W}\boldsymbol{x}^i)$$

- We can rewrite this in matrix form:

$$\boldsymbol{x}^{i+1} = (\boldsymbol{I} - \alpha \boldsymbol{W}^T \boldsymbol{W})\boldsymbol{x}^i + \alpha \boldsymbol{W}^T \boldsymbol{p}$$

- This is a recurrence relation, with solution for iteration $n$:

$$\boldsymbol{x}^n = \boldsymbol{A}^n \boldsymbol{x}^0 + \alpha \left[ \sum_{k=0}^{n-1} \boldsymbol{A}^k \right] \boldsymbol{W}^T \boldsymbol{p}, \quad \boldsymbol{A} = (\boldsymbol{I} - \alpha \boldsymbol{W}^T \boldsymbol{W})$$

# Approximate algebraic method

- We have rewritten the SIRT equation to:

$$\boldsymbol{x}^n = \alpha \left[ \sum_{k=0}^{n-1} \boldsymbol{A}^k \right] \boldsymbol{W}^T \boldsymbol{p}$$

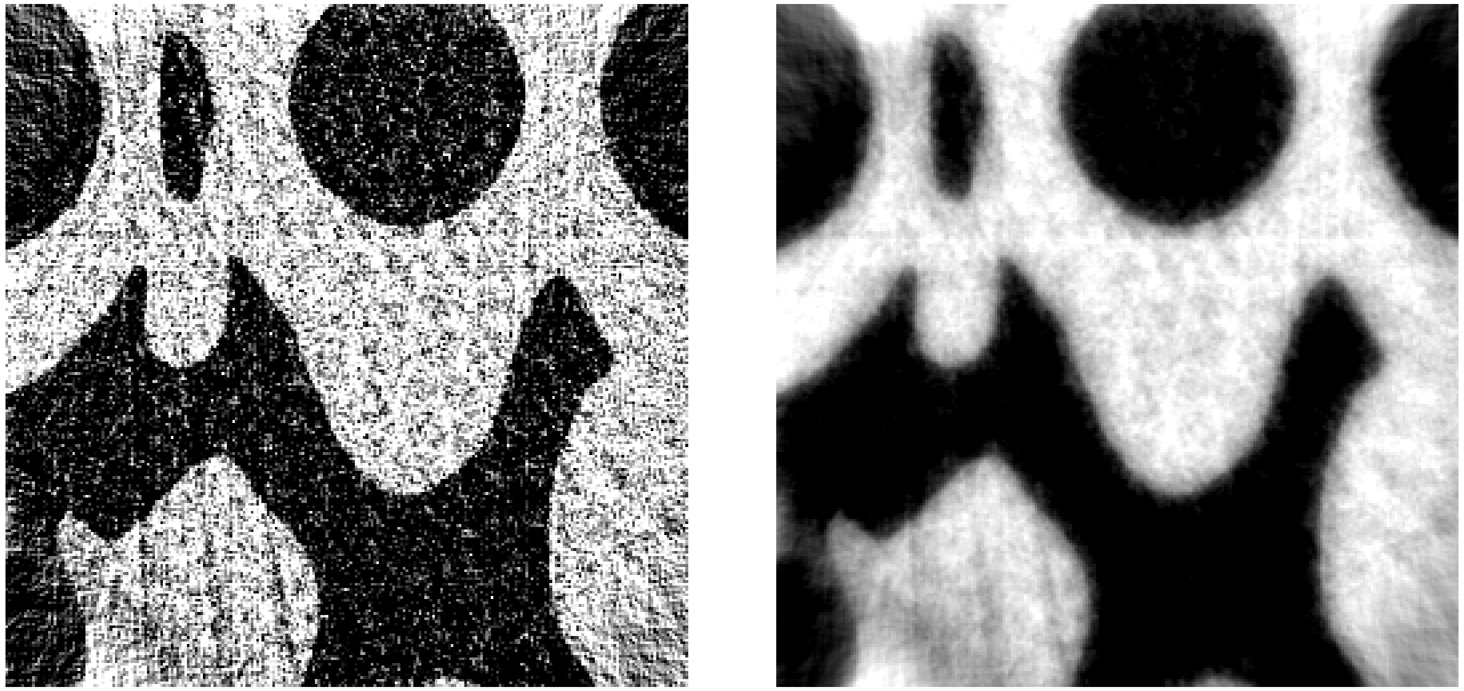
- Compare with the “backproject, then filter” FBP equation:

$$FBP(\boldsymbol{p}, \boldsymbol{h}') = \boldsymbol{C}_{\boldsymbol{h}'} \boldsymbol{W}^T \boldsymbol{p}$$

- Approximate the $\boldsymbol{A}^k$ sum with a convolution
  - Filter can be precalculated for a certain acquisition geometry
- Computation time of reconstruction is identical to FBP

# Filter based methods

- Filter-based methods can be applied to ROI tomography by standard padding
- An example of the MR-FBP method: (Pelt & Batenburg, 2014)

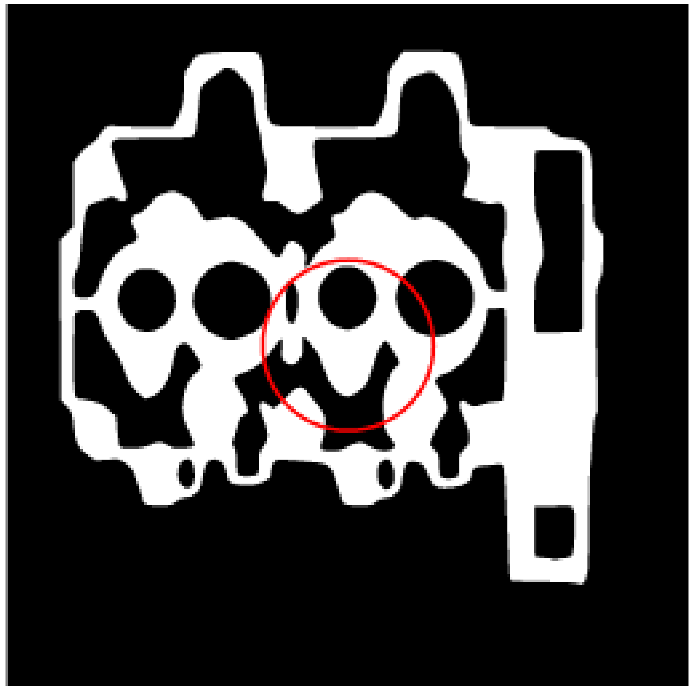


(a) FBP (b) MR-FBP

- Proposal: extend these methods to exploit prior knowledge

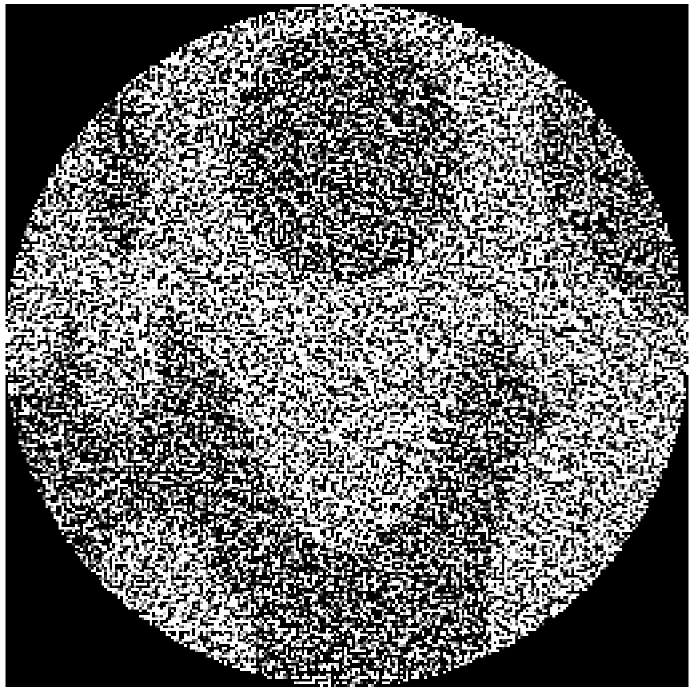
# Preliminary results

1024 detectors truncated to 256, 128 projections, Poisson noise

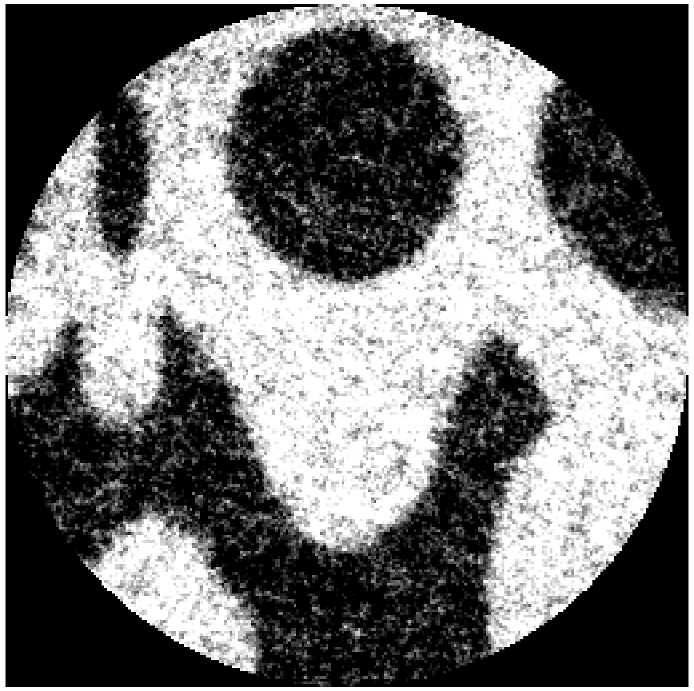


(a) Phantom

(b) Phantom



(c) FBP



(d) SIRT-FBP

# Approach

- Many prior-based methods use SIRT (Landweber iteration)
  - FISTA (TV-MIN, wavelets), box constraints on pixel value
- Single iteration can be written as two steps:

$$\boldsymbol{x}^{k+1} = S(\boldsymbol{x}^k) + \boldsymbol{dx}^{k+1}$$

- Split reconstruction at iteration $k$ into SIRT part and prior part:

$$\boldsymbol{x}^k = \boldsymbol{x}_s^k + \boldsymbol{y}^k$$

- Using definition of SIRT, we can show ($\boldsymbol{A} = \boldsymbol{I} - \alpha \boldsymbol{W}^T \boldsymbol{W}$):

$$\boldsymbol{x}^{k+1} = S(\boldsymbol{x}_s^k) + \boldsymbol{A}\boldsymbol{y}^k + \boldsymbol{dx}^{k+1}$$

# Approach

$$
\begin{aligned}
\boldsymbol{x}^{k+1} &= S(\boldsymbol{x}_s^k) + \boldsymbol{A}\boldsymbol{y}^k + \boldsymbol{d}\boldsymbol{x}^{k+1} \\
\boldsymbol{x}_s^{k+1} &= S(\boldsymbol{x}_s^k) \\
\boldsymbol{y}^{k+1} &= \boldsymbol{A}\boldsymbol{y}^k + \boldsymbol{d}\boldsymbol{x}^{k+1}
\end{aligned}
$$

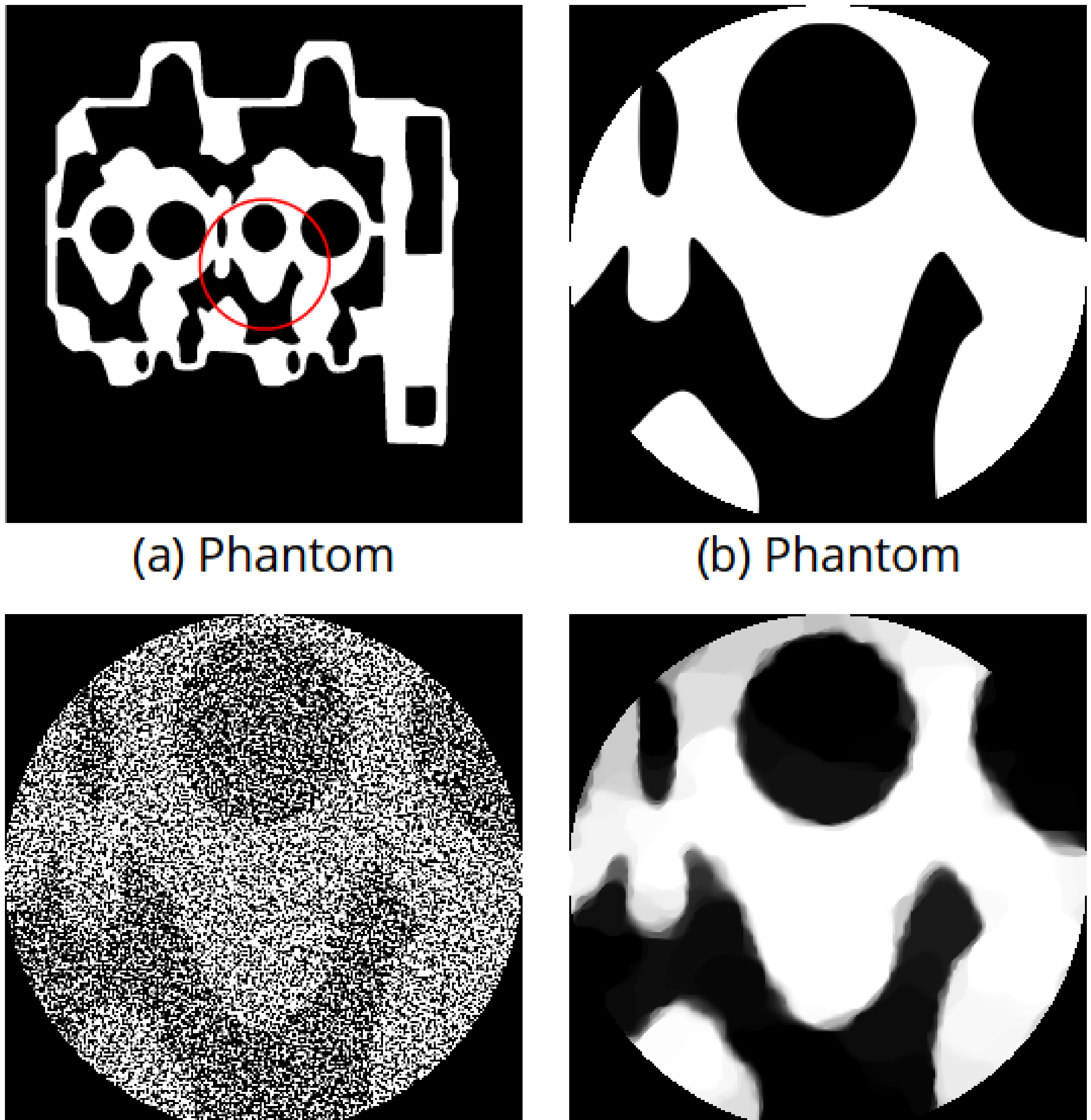
- Idea: only compute $\boldsymbol{x}_s^{k+1}$ and $\boldsymbol{y}^{k+1}$ inside ROI
- $\boldsymbol{x}_s^{k+1}$ can be approximated inside ROI by FBP with a special filter (+ padding) (Pelt & Batenburg, to appear 2015)
- $\boldsymbol{y}^{k+1}$ can be calculated by the prior-based method, only applied inside ROI

# Analysis

- Only volume inside ROI is needed during reconstruction
- Computation time is identical to prior-based method inside ROI + 1 FBP every iteration
- Three approximations are made:
  - SIRT is approximated by FBP with a special filter
  - The prior knowledge is only applied inside ROI
  - The effect of the prior on pixels outside the ROI is ignored

# Preliminary results

1024 detectors truncated to 256, 128 projections, Poisson noise



(a) Phantom (b) Phantom

(c) FBP (d) Local TV-MIN

# Conclusion

- We have a filter-based method to approximate a prior-based method inside a ROI
- Goals:
    - See how well the method works on real-world data
    - Find interesting datasets to apply it on
    - Try different priors, datasets, and ROI sizes
    - Optimize parameters and padding to improve quality


References:

[1] Pelt, D. M., & Batenburg, K. J. (2015). Accurately approximating algebraic tomographic reconstruction by filtered backprojection. To appear in *Proceedings of the 2015 International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine.*

[2] Pelt, D. M., & Batenburg, K. J. (2014). Improving Filtered Backprojection Reconstruction by Data-Dependent Filtering. *Image Processing, IEEE Transactions on, 23*(11), 4750-4762.

# Appendix: Proposed method

- Calculation of iteration:

$$S(\boldsymbol{x}^k) = \boldsymbol{A}\left(\boldsymbol{x}_s^k + \boldsymbol{y}^k\right) + \alpha \boldsymbol{W}^T \boldsymbol{p}$$

$$= \boldsymbol{A}\boldsymbol{x}_s^k + \alpha \boldsymbol{W}^T \boldsymbol{p} + \boldsymbol{A}\boldsymbol{y}^k$$

$$= S(\boldsymbol{x}_s^k) + \boldsymbol{A}\boldsymbol{y}^k$$

# Appendix: Proposed method

- Calculation of $\boldsymbol{A}\boldsymbol{y}^k$:

$$
\begin{aligned}
\boldsymbol{A}\boldsymbol{y}^k &= (\boldsymbol{I} - \alpha \boldsymbol{W}^T \boldsymbol{W})\boldsymbol{y}^k \\
&= \boldsymbol{y}^k - \alpha(\boldsymbol{W}_{\mathcal{L}}^T + \boldsymbol{W}_{\mathcal{O}}^T)\boldsymbol{W}\boldsymbol{y}^k \\
&= \boldsymbol{y}^k - \alpha \boldsymbol{W}_{\mathcal{L}}^T \boldsymbol{W}\boldsymbol{y}^k - \alpha \boldsymbol{W}_{\mathcal{O}}^T \boldsymbol{W}\boldsymbol{y}^k \\
&\approx \boldsymbol{y}^k - \alpha \boldsymbol{W}_{\mathcal{L}}^T \boldsymbol{W}_{\mathcal{L}}\boldsymbol{y}^k
\end{aligned}
$$